Lagrange Multiplier Embedded Mesh Method

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Our objective is to permit more effective simulation of complex phenomena by using multiple, but more easily generated, meshes. Figure 1 shows a simple example where a fluid is flowing past a moving solid mesh. The two meshes could in fact be processed by two distinct simulation codes, using meshes that enhance their respective robustness. This general class of method is known as embedded mesh. A number of different approaches exist, yet many have not become popular due to a variety of side effects. Most of these approaches have also focused on coupling finite volume grids. In our project, the focus is on the coupling of Arbitrary Lagrangian-Eulerian (ALE) type finite element meshes without the use of an overset grid. Close attention is paid to the accuracy and efficiency to make our method practical. The new methods are currently being implemented in our new FEusion software library and will be modular and documented for potential utilization in other LLNL codes.

Project Goals

We seek a general software tool to interface embedded meshes. The formulation as implemented should accommodate the computational efficiency expected of an explicit time integration method. It should also be extensible to different physics and finite element discretizations. Two model problems are the focus:

1. Fluid structure interaction of solid and fluid meshes. An example would be a Lagrange shell mesh subjected to an air blast on an ALE or Eulerian mesh (Fig. 2). Our approach would obviate the need for conformal meshing of the fluid domain to the structural geometry and potentially be more robust.
2. Coupled solid-electrodynamics of a moving conductor in an air background grid (Fig. 3). Applications of this phenomenon would be the rail gun or flux compression generator.

Relevance to LLNL Mission

The methodology developed in this project will enhance our computational tools, permitting more robust simulation of complex, real-world engineering models in support of many LLNL programs.

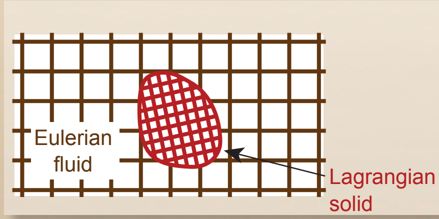

Figure 1. Schematic application of an embedded grid method to model the interaction of a solid body with a surrounding fluid. The software couples the fluid and solid along their common boundary.

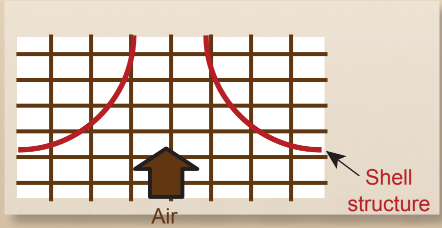

Figure 2. Blast loading on a shell structure.

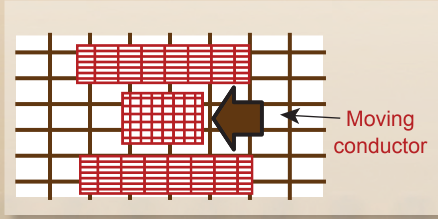

Figure 3. Moving conductor on an air background grid.

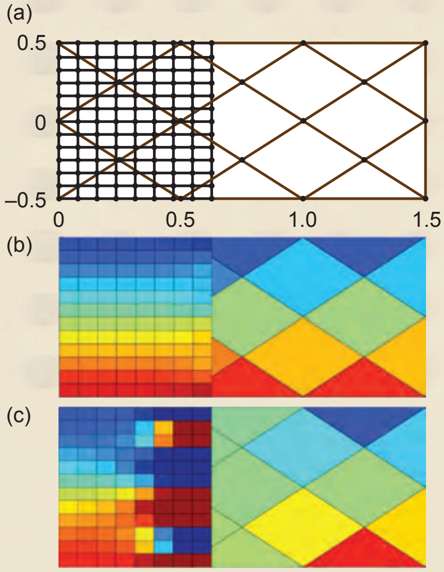

Figure 4. **(a) Overlapping fine (black) and coarse (brown) meshes. (b) Stress at interface with a soft material on fine and coarse meshes. (c) Stress at interface with stiff material on fine mesh and soft material on coarse mesh.**

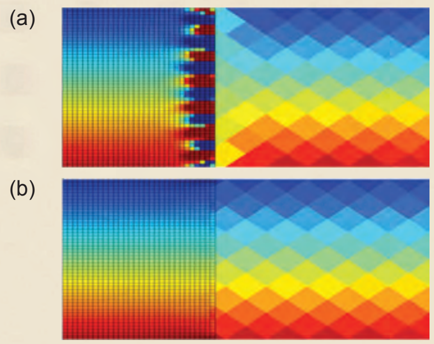

Figure 5. **Stress at interface with stiff material on fine mesh and soft material on coarse mesh: (a) using standard approach, (b) using new modified approach.**

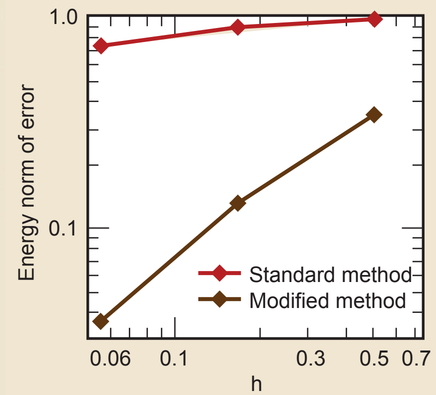

Figure 6. **Energy norm of error vs. element size for stiff material on fine mesh and soft material on coarse mesh. Modified method converges at optimal rate.**

FY2009 Accomplishments and Results

As year one of the project, initial development was begun for the FEusion embedded mesh coupling software. The data model accepts a superposed mesh (*e.g.*, Lagrange solid) and a background mesh (*e.g.*, ALE fluid). The software identifies overlaps, resulting voids on the background grid, and the strains on the "cut" background cells. It then generates constraint equations that enforce velocity continuity and associated transformation operators along the boundary between the two meshes. An interface to LLNL's ALE3D code was created and a developers guide written detailing the specific data structures exchanged with FEusion. A MATLAB implementation was also developed as a numerical test bed for the new coupling methods.

Most of the formulation developed so far has been for fluid structure interaction. A Lagrange multiplier is defined on the solid surface mesh and used to enforce velocity continuity between the fluid and solid. A special dual Lagrange multiplier yields efficient monolithic coupling. Typical model applications will have a background fluid grid that is finer than the superposed solid mesh because of the needed resolution.

Our MATLAB results in Fig. 4 (b) show that stress is smoothly captured across the fluid and solid boundaries for this simple example. Nonetheless, some models will have a coarse fluid mesh and this can result in so-called mesh locking [Fig. 4(c)]. This known pathology of many embedded grid methods can be avoided using an intermediate mesh. Figure 5 shows a finer mesh representation of the stiff and soft material problem with results from the original and modified approaches. The new modified approach eliminates the stress oscillations. Optimal convergence is documented in Fig. 6.

Related Reference

Puso, M. A., "A 3-D Mortar Method for Solid Mechanics," *International Journal for Numerical Methods in Engineering*, **59**, pp. 601–629, 2004.

FY2010 Proposed Work

Our proposed FY2010 work is as follows:

1. **Interface FEusion with ALE3D.**
2. **Extend application to handle shell solid models.**
3. **Modify advection routines in ALE3D to accommodate "cut" fluid cells.**
4. **Interface FEusion to Diablo for solid-electrodynamics problem.**